Resonant Ultrasound Spectroscopy for arbitrarily shaped samples using Finite Element Method and Levenberg-Marquart algorithm

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ABSTRACT

Resonant Ultrasound Spectroscopy (RUS) is a method that uses experimental and numerical techniques. It as been set up in the 1990's while calculation speed of computers increases. It aims to characterize the elastic moduli of a material sample by matching experimental sets of resonance frequencies with calculated ones and, by resolving an inverse problem, get the elastic moduli of the sample. RUS processing usually uses the Rayleigh-Ritz method to compute the eigenfrequencies. Levenberg-Marquart's (LM) algorithm is applied usually in order to minimise the relative error between experimental and numerical eigenfrequencies. In order to apply to arbitrarily shaped samples the Finite Element Method (FEM) must be used because the Rayleigh-Ritz method is efficient only for simple parameterizable shapes. Here FEM is used to calculate the numerical set of frequencies thanks to Code_Aster software. Matlab© is used to script the different operations : create mesh, launch the FEM calculation and solve inverse problem using LM algorithm. The applicability of this modified RUS method is verified by testing all the calculation chain with an isotropic cylindrical sample of copper, so the elastic tensor is defined by the two Lamé parameters.

INTRODUCTION

The knowledge of the mechanical moduli of a material can be very important in many situations. In structural mechanics of course but also in medical applications. Indeed, by knowing mechanical moduli of somebody's bones it is possible to deduce the health of his skeleton. A traditional way to get the mechanical moduli is to realise a tensile test. But it is not possible to realise such a test on little objects such as a mouse teeth. That's where vibration analysis is used. Indeed we only need to get the frequency response of the sample to obtain the mechanical moduli thanks to the Resonant Ultrasound Spectroscopy method.

The first work on this subject was made by Migliori *et al.* [4] in 1993 and RUS was formalized by Maynard [3] in 1996. At this time, only common shaped samples are used for RUS. Indeed, the numerical set of frequency was calculated thanks to the Rayleigh-Ritz method which is only available for simple shaped materials. In 2004 FEM is applied to RUS by Plesek *et al.*[5] using Fixed Point Iteration method (FPI).

The objective of our works is to design and validate a numerical code to solve the FEM-RUS optimisation problem with Levenberg-Marquart (LM) algorithm. The used of the optimisation toolbox of Matlab is necessary, that's why we need to realise a coupling between Matlab and Code_Aster. The optimisation by LM will be tested and validated. As we aim to know the elastic tensor of real objects and samples, the applicability of this method for a short calculation time is studied.

METHOD

The RUS consists in three parts. First, the frequency response of the sample is measured and the eigenfrequencies obtained. Then an equivalent FEM model is created and the numerical eigenfrequencies can be calculated. Finally the minimisation of the relative error between the two set of frequencies is realised. All this experimental/numerical method is shown schematically on figure 1.



Figure 1: Schematic description of the resonant ultrasound spectroscopy method

The aim of this work is to verify the feasibility of such a method, that is why we will determine the elastic tensor of a cylinder of copper, with a radius of 4,98 mm and 9,5 mm high : an isotropic material.

Experiment

In our study the frequency response of the sample is obtained by placing it between two transducers : an emitter and a receiver as shown in figure 2. The sample is excited between 110 kHz and 350 kHz and the frequency response is then obtained by calculating the ratio between the excitation spectra and the received signal spectra. In order to measure this frequency response we use a signal generator that generate very short sine for discrete frequencies (Bode 100, Omicron electronics GmbH, Klaus, Austria). The receivers output signal is preconditioned befor recording by a broadband charge amplifier (HQA-15M-10T, Femto Messtechnik GmbH, Berlin, Germany).

Once the frequency response is obtained we need to detect the eigenfrequencies. For this purpose, a pic detection is done. The axial symmetry (cylindrical sample) of our sample induce double frequencies, it means two modes that have very close eigenfrequencies. It means that for some frequencies two different modes can be excited. We chose here to circumvent the problem of matching double frequencies by keeping only one frequency, the average of the two. This choice is motivated by the fact that, whether the FEM would always calculate those double frequencies, the experiment could not always detect them because of the polarization of the transducers and, so, the matching would be very difficult.



Figure 2: Photograph of the experiment, cylinder of copper between the emitter and receiver, supposed with free boundary conditions.

Optimisation

We introduce here the optimisation problem. In our study we need to match two set of frequencies. For this purpose we will need to minimise a cost function F defined by the sum over the frequency of the squared relative error between experimental calculated resonant frequency and numerical calculated ones, so :

$$F = \sum_{i} \left| \frac{|f_i^{exp} - f_i^{num}|}{f_i^{exp}} \right|^2, \tag{1}$$

where f_i^{exp} is the ith experimental eigenfrequency and f_i^{num} the numerical one.

The optimisation process is done using the Levenberg -Marquardt algorithm in Matlab. It can deal with multiple variables and non-linear problem. It is a very smart iterative algorithm that use the advantages of the two major algorithms of matrix inversion : the Gauss-Newton method and the gradient method. The first one is calculating the steepest descent while the second aims to linearise F near to its minimum. For every iteration the two algorithm calculate the n+1 term by :

$$\begin{cases} x_{n+1} = x_n - A^{-1}B, & \text{for G-N method,} \\ x_{n+1} = x_n - D^{-1}B, & \text{for gradient method,} \end{cases}$$
(2)

where A is the Jacobian matrix of the cost function, D is diagonal matrix and B the gradient vector of F.

The Levenberg-Marquardt algorithm aims to combine those two algorithms by calculating :

$$x_{n+1} = x_n - (A + \lambda_{LM}D)^{-1}B.$$
 (3)

The λ_{LM} parameter defines the relative importance of the two different methods in the solving process, it is called the damping factor. This combination of two well known algorithms explain why it is called a "smart algorithm". The algorithm determines the optimum value of the λ_{LM} parameter for each iteration. In fact λ_{LM} is about 0,01 at the first iteration and it's reducing step by step. So first, the algorithm is more taking care of the steepest descent and then of the little variations of the cost function. This is very powerful because it means that, if there is only one local minimum of the function, the minimization will be achieved with a few iterations even if the starting point of iterations is far from minimum.

It is important to notice that for each iteration of the minimization problem this algorithm needs three numerical calculation in order to calculate the 2D derivative of the cost function by finite difference, which influences the computation time of this method. When using Rayleigh-Ritz method, the derivative are readily available, which is not the case with the FEM.

Bernard *et al.* [2] used sixteen frequencies to estimate the nine component of the elastic tensor for an anisotropic material. Knowing that, we use only the nine first frequencies of our sample (experimental and numerical) for the optimisation because we only have to determine λ and μ , the two Lamé parameters.

Coding FEM and LM

All the interest of this paper is to interface a FEM software with an optimisation algorithm in order to create a fully functional program that can, knowing the frequency response of a sample, determine precisely and quickly its elastic tensor.

As we already discuss it is Code_Aster that is chosen as FEM software in this work, it is an EDF developed software. Indeed it has a non-negligible advantage : it can be used only with command lines which is the best for our goal to control all the process with one and only program in Matlab. This way we can manage a whole chain of instruction such as lunch the generation of the mesh (by GMSH), start the calculation with Code_Aster ,read the text formatted result files and do the optimisation in one program. We recall that Code_Aster can calculate rather the modes shape and eigenfrequencies.

One of the function called by the program aims to make the mesh that will be used for the FEM with the specify cylinder dimension and with specific characteristic length (CL) of the mesh that is needed. An example of used mesh is shown in figure 3. Other functions create all the files Code_Aster needs to run the calculation. For example, one of the functions has to write a text file and change the Young modulus and Poisson's ratio (coefficients use by Code_Aster linked to λ of our material automatically, that is important for the optimisation to work.

We are now interested of estimating the better ratio time of calculation / relative error for our FEM calProceedings of Acoustical Engineering Projects 2014



Figure 3: Mesh of our 9,54 mm high copper sample for a characteristic length of mesh CL = 1 mm

culation. That's why we compare our first frequencies with resonant frequencies calculated with Ravleigh-Ritz method. The results are available on figure 4. In this study the mesh is constituted of quadratic elements in order to increase the computation speed and decrease the characteristic length of the mesh. As we can see on this results from a characteristic length (CL) of mesh of 1 mm (= 9 points/ λ_{min}) the relative error is quite acceptable (0.015 % for the first frequency and up to 0.1 % for the last frequency we are going to use to resolve the inverse problem). By seeing our results we could see that eleven points per wavelength would be better but in term of calculation time its not possible. Indeed it multiplies by 2,5 the time of calculation compared to 9points/ λ_{min} calculation which already takes about 35 seconds. The minimum wavelength used λ_{min} is defined with the second Lamé Parameter μ , the density ρ and the maximum frequency of our study $f_{max} = 250 \,\text{Hz}$:

$$\lambda_{min} = \frac{c_{min}}{f_{max}} \quad \text{with} \quad c_{min} = \sqrt{\frac{\mu}{\rho}}.$$
 (4)



Figure 4: Relative error between Rayleigh-Ritz calculated frenquency and Finite Element Method calculation in function of the number of points by minimum wavelength (maximum frequency calculated is 250 kHz)

We can see on figure 5 an example of experimental frequency response compared to the results of the FEM code for standard elastic moduli values of copper. It is clear that the magnitude of the numerical calculated frequencies is good, but their are some differences. Indeed, the aim of this method is to quantify the variations to the theoretical values in order to be able to detect some specific modifications and defaults of the material.



Figure 5: Experimental frequency response of the copper specimen (in blue line) detected pics (in red circles) and Code_Aster calculated frequencies (in black disks) for standard elastic moduli values of copper[1]

In order to solve the inverse problem we use the lsqnonlin Matlab function which minimises the cost function Fwith LM algorithm. It's a matter of fact, the algorithm needs the local derivatives for each iteration, which are calculated by finite differences. The space between the points where the cost function is estimated by the algorithm in order to calculate the derivative is really tiny. Based on this observations we suppose that the input parameters of the function have to be normalised so that the derivative is not estimated to zero and the iteration process can lead to a coherent result. This optimisation is done over two coefficients : C_{11} and C_{44} . Those to coefficients are the components of the elastic tensor, they can easily be linked to the Lamé parameters with :

$$C_{44} = \mu, \quad C_{11} = \lambda + 2\mu.$$
 (5)

RESULTS

The results and computation time presented bellow have been calculated in a cluster with Intel® Xeon® E5620 processors clocked at 2.40GHz. For our work, no computational parallelization is done and only one processor and 2,5 GB of Random-access memory (RAM) is used when CL = 1 mm.

In order to control and verify the optimisation process we decided to calculate the cost function F for multiple combinations of C_{11} and C_{44} . The figure 6 shows the calculated "map". Moreover we also plot the (C_{11}, C_{44}) couple use by the optimisation process (red lines).

The elastic moduli obtained during this optimisation are compared to "classical"[1] ones in table 1.



Figure 6: Cost function F for C_{11} and C_{44} variations and optimisation example with stating point (in green diamon), result point (in white diamon) and the algorithm iteration points. At final point F = 0.027 %.

	C_{11} (GPa)	C_{44} (GPa)
Classical values	$189,\!65$	48,12
RUS calculated values	201,13	47,61

Table 1: RUS calculated values of elastic moduli of the cylinder of copper compared to the common used values of copper in GPa

DISCUSSION

The figure 6 carries two informations. First, we can see that C_{44} is the parameter that is most influential in low frequency (our case here, only the nine first frequency are used). Indeed the very tiny variation of the cost function with C_{44} involve a huge error on the measure contrary to C_{11} . The second information is the (C_{11}, C_{44}) couple values taken during the iteration. In this example five iterations are needed to get to the minimum point (including calculation at the first point). For this optimisation, 10 minutes are needed to get to the minimum. In term of stopping parameters, we tested the optimisation with a very low tolerance on the residual and the algorithm stopped when its about $2,7.10^{-4}$ because of non-further progress possible. So, the stopping parameter is taken not too tiny to avoid non-useful calculations.

The table 1 shows that our results are fully coherent with classical elastic moduli with a little difference induced by the variability of making the material. Moreover, we can estimate the error on the obtained moduli. Indeed, the value of the cost function at final point of optimisation is 0,027% and the convergence test indicates that the sum of the squared errors (over the 9 first frequencies) is about 0,02%. So the real error on the estimated values is 0,047%.

We have presented in this paper results that are obtained by an all chain of instructions. We managed to control those instructions with one and only program that create the good mesh with quadratic or linear element and with the wanted dimensions. Then it create the Code_Aster command files with the specific mechanical values wanted and launch them all. Finally, the results are read from the text file. This chain of commanding Code_Aster and GMSH is then included in the the calculation of the cost function and in the optimisation process.

According to the results we presented in this paper we believe we have validated this elastic moduli characterization process with a simple sample. But this study can easily be upgraded to manage anisotropic materials. Indeed only more frequencies have to be considered and a few lines of the Code_Aster command file have to be changed.

We believe that this way of using RUS with FEM and LM algorithm is fully usable for any little arbitrarily shaped anisotropic sample, by creating a equivalent mesh, with a totally reasonable computation time.

REFERENCES

- [1] "Propriétés des matériaux" http://www. simulationmateriaux.com, 2014.
- [2] S. BERNARD, Q. GRIMAL et P. LAUGIER "Accurate measurement of cortical bone elasticity tensor with resonant ultrasound spectroscopy", *Journal of the Mechanical Behavior of Biomedical Materials* (2012).
- [3] J. MAYNARD "Resonant ultrasound sepctroscopy", *Phys. Today* (1996), p. 26–31.
- [4] A. MIGLIORI, J. SARRAO, W. M. VISSCHER, T. BELL, M. LEI, Z. FISK1 et R. LEISURE – "Resonant ultrasound spectroscopic techniques for measurement of the elastic moduli of solids", *Physica B* (1993).
- [5] J. PLESEK, R. KOLMAN et M. LANDA "Using finite element methode for the determination of elastic moduli by resonant ultrasound sepctroscopy", *Journal of the Acoustical Society of America* (2004).