Resonant Ultrasound Spectroscopy for arbitrarily shaped samples using Finite Element Method and Levenberg-Marquart algorithm
Validation with isotropic material

Raphaël LEIBA

University Pierre et Marie Curie - Paris VI

February 26, 2014

Project tutored by Quentin GRIMAL
The knowledge of the elastic tensor of a sample gives informations about the health of the material.

Usual way to get it is to realise a tensile test. We can’t have all the tensor components in one test.
Introduction: Elastic tensor

- The knowledge of the elastic tensor of a sample gives informations about the health of the material.
- Usual way to get it is to realise a tensile test. We can’t have all the tensor components in one test.
- Very complicated with tiny objects or with anisotropic materials
- Example of orthotropic materials:

\[
C_{ij} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}.
\] (1)

Validation with isotropic materials:

\[
C_{ij} = \begin{bmatrix}
\lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & \lambda + 2\mu & 0 & 0 \\
C_{44} & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}.
\] (2)

\(C_{11} = \lambda + 2\mu\) and \(C_{44} = \mu\).
Introduction: Elastic tensor

- The knowledge of the elastic tensor of a sample gives informations about the health of the material.
- Usual way to get it is to realise a tensile test. We can’t have all the tensor components in one test.
- Very complicated with tiny objects or with anisotropic materials
- Validation with isotropic materials:

\[
C_{ij} = \begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}.
\]  

with \(C_{11} = \lambda + 2\mu\) and \(C_{44} = \mu\).
Introduction : RUS

- Stands for Resonant Ultrasound Spectroscopy
- Aims to characterise the Elastic Tensor knowing the frequency response of a sample.
- Dimensions of the sample $\Rightarrow$ frequency domain of ultrasound
RUS : Steps of an Experimental/numerical method

Experiment

Specimen

Acquisition system

Generator

Emitter

Reciever

Resonance frequencies $f_{\text{exp}}$

Pic detection

Resonance frequencies $f_{\text{num}}$

Numerical calculation with finite element method

Elastic moduli of ideal sample

Cost function $F(f_{\text{exp}}, f_{\text{num}})$

Minimisation of $F$

Elastic moduli of the specimen
The RUS method were invented in the 1990’s thanks to the increase of computation power of computers. Different evolutions:

- **Migliori et al. (1993)**: *Resonant ultrasound spectroscopic techniques for measurement of the elastic moduli of solids*  
  - Basis of the method

- **Maynard (1996)**: *Resonant ultrasound spectroscopy*  
  - formalization of RUS (Rayleigh-Ritz method and Levenberg-Marquart (LM) scheme).

- **Plesek et al. (2004) and Liu et al. (2011)**  
  - FEM is fully suitable with RUS
The RUS method were invented in the 1990’s thanks to the increase of computation power of computers. Different evolutions:

- **Migliori et al. (1993)**: *Resonant ultrasound spectroscopic techniques for measurement of the elastic moduli of solids*
  - Basis of the method
- **Maynard (1996)**: *Resonant ultrasound spectroscopy*
  - Formalization of RUS (Rayleigh-Ritz method and Levenberg-Marquart (LM) scheme).
- **Plesek et al. (2004) and Liu et al. (2011)**
  - FEM is fully suitable with RUS

**Our goal**

RUS for arbitrarily shape samples using FEM and LM scheme
In this study, the resonant frequency are calculated by finite element method (FEM) thanks to Code_Aster software.

Mesh are generated by GMSH.

Code_Aster and GMSH can both be used with a graphical interface or with command lines.

Here, the commands lines are used in order to launch those software in chain.

In addition, they are free software.

Example of Code_Aster command file

```
COPPER=DEFI\_MATERIAU(ELAS=\_F(E= 128e9 ,
        NU= 0.33 ,
        RHO= 9.27e3 ,),);
```

This is the part of code which is modified during the inverse problem.
Finite Element Method (2)

- We want the best computation time/error ratio \( \Rightarrow \) convergence test.
- It is done in function of the smallest wavelength used: the one of the transversal wave for maximum frequency (250 kHz)

Mesh example, CL = 1mm

Convergence Test

Application for a cylinder of cooper radius : 4.98 mm, height : 9.5 mm
Optimisation : Levenberg-Marquart algorithm

- The Levenberg-Marquart scheme is based on combining two minimisation algorithms: Gauss-Newton and gradient method.
- The gradient method is using the steepest descent to get to the next iteration.
- The Newton-Gauss method aims to linearise the function. The next point is the zero of the linearised function.
- The Levenberg-Marquardt algorithm aims to combine those two algorithms: it mainly uses the gradient method first and then the Gauss-Newton method to be more precise.
- The cost Function $F$ is defined by:

$$
F = \sum_{i} \left| \frac{f_{i}^{\text{exp}} - f_{i}^{\text{num}}}{f_{i}^{\text{exp}}} \right|^2,
$$

where $f_{i}^{\text{exp}}$ is the $i$th experimental eigenfrequency and $f_{i}^{\text{num}}$ the numerical one.
Matlab is defined as our central script in order to use the optimisation toolbox \( \Rightarrow \text{lsqnonlin} \) function uses LM algorithm.

Both GMSH and Code_Aster are launched from Matlab.

In order to use \texttt{lsqnonlin} properly, all the files that Code_Aster uses are modified by Matlab functions, such as the example previously seen.

For our first works no parallel computation have been made. It is mostly Matlab that is restrictive.
Matlab: One and only program for RUS - Scheme

Mesh information file generation → GMSH → Mesh → FEM Calculation with Code_Aster → Cost Function $F$

Load experimental frequency response & get eigenfrequencies

Command file generation with elastic moduli informations

Minimisation process, while $F > F_{\text{min}}$

if $F = F_{\text{min}}$

Elastic moduli of the sample
Matlab: One and only program for RUS - Scheme

Mesh information file generation → GMSH → Mesh → FEM Calculation with Code_Aster → Cost Function $F$

Minimisation process, while $F > F_{\text{min}}$

Command file generation with elastic moduli informations → Load experimental frequency response & get eigenfrequencies

Cost Function $F$ → if $F = F_{\text{min}}$

Elastic moduli of the sample

Computation is realised on a cluster with processors cadenced at 2.4GHz and 42GB of RAM. Only one processor and 2.5GB of RAM are used.
Results - Cylinder of copper

**Figure**: Cost function $F$ for $C_{11}$ and $C_{44}$ variations and optimisation example with stating point (in green diamon), result point (in white diamon) and the algorithm iteration points. At final point $F = 0.027\%$. 
**Example of FEM results - Optimised moduli**

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>118,984</td>
</tr>
<tr>
<td>2</td>
<td>142,333</td>
</tr>
<tr>
<td>3</td>
<td>142,336</td>
</tr>
<tr>
<td>4</td>
<td>153,276</td>
</tr>
<tr>
<td>5</td>
<td>153,277</td>
</tr>
<tr>
<td>6</td>
<td>159,649</td>
</tr>
</tbody>
</table>

- Pair the frequencies is not obvious. Here we pair the two set of frequencies by increasing order. It would be better to pair the frequencies considering the modal shapes.
- It is important to notice that the minimisation process only works when the elastic moduli are normalised.
Discussions

- For this example, the total error on the estimated moduli is 0.047% (value of $F$ at final point and error estimated with the convergence test).
- During this project an all chain of programs have been created in Matlab in order to:
  - make the mesh,
  - create the command files for Code_Aster,
  - launch the FEM Computation and get back the resulting frequencies,
  - pair the two set of frequencies and calculate the cost function $F$.
- Regarding to our results we assume that we have validated this elastic moduli characterization process with a simple sample.
- RUS for arbitrarily shaped anisotropic material is an ambitious project in regards of it’s complexity.
- In this study, in order to validate and simplify the method we used simple shaped and isotropic samples (cylinder).
This study can easily be upgraded to manage anisotropic materials:
- more frequencies have to be considered and a few lines of the Code_Aster command file have to be changed.
- Of course the computation time will increase because of the $N$ number of variables and the calculation of $N$-D derivatives.

Parallel the computation of derivatives could be implemented decreasing computation time of each iteration.

In case of arbitrarily shaped samples the mesh is not so easy to generate: a scanner has to be used to know the precise shape.